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The Refraction of a Plane Shock Wave at an Air-Water Interface

RICHARD A. SKOP and J. GORDAN SHOWALTER

Applied Mechanics Branch
Ocean Technology Division

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The normal refraction at an air water interface of a plane shock wave incident through the air is considered. The perfect gas and Tait equations of state are used, respectively, to represent the thermodynamic properties of the air and water. For refracted pressures up to 1200 atmospheres, the calculations reveal that the in-water shock front and particle velocities and the changes in density differ by less than 10% from the in-water shock front and particle velocities and the changes in density predicted by acoustic theory. The calculations also show that, for

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20. Abstract (Continued)

a given incident shock, the ratio of the refracted pressure to the pressure reflected from a rigid wall remains above 96% for reflected pressures up to 1200 atmospheres.

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CONTENTS

THE REFRACTION OF A PLANE SHOCK WAVE AT AN
AIR-WATER INTERFACE

INTRODUCTION

The normal refraction at an air-water interface of a plane shock wave incident through the air is investigated. The purposes of this investigation are twofold:

a. To define under what conditions the in-water shock front and particle velocities and the changes in density can be predicted by acoustic theory; and,

b. To determine under what conditions the refracted pressure can be calculated by treating the interface as a rigid wall.

Jump Conditions Across A Shock

The jump conditions across a shock express the conservations of mass, momentum, and energy in a control volume enclosing the shock. From a reference frame moving with the shock, these conditions are given by [1]

$$\rho_a v_a = \rho_b v_b \tag{1a}$$

$$P_a + \rho_a v_a^2 = P_b + \rho_b v_b^2 \tag{1b}$$

$$e_a + P_a/\rho_a + v_a^2/2 = e_b + P_b/\rho_b + v_b^2/2 \tag{1c}$$

Here, the subscripts "a" and "b" refer to ahead of and behind the shock, respectively. The symbols ρ, P, e, and v denote, in order, the fluid density, pressure, specific internal energy, and relative velocity with respect to the shock.

The former three of these quantities cannot vary independently across the shock but are constrained to satisfy the equation of state of the fluid. For air, the perfect gas equation of state yields the relationship

$$e = P/[(\gamma-1)\rho] \tag{2}$$

where γ is the ratio of specific heats. For water, the constraint is expressed by the Tait equation of state [2]

$$P = P_o + B[(\rho/\rho_{wo})^n - 1] \tag{3a}$$

Here, ρ_{wo} is the water density at atmospheric pressure P_o, n is a

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dimensionless constant, and B is a second constant defined by

$$B = \rho_{wo} \, C_{wo}^2 \, /n \tag{3b}$$

where C_{wo} is the speed of sound in water under atmospheric conditions.

Mechanical Shock Conditions

Equation (1a) gives the condition

$$V_a = \eta \, V_b \tag{4}$$

where the shock compressibility factor η is defined by

$$\eta = \rho_b/\rho_a \tag{5}$$

Upon substitution of equation (4), equation (1b) yields

$$V_b^2 = (P_a/\rho_a) \ (y-1) \ [1/\eta(\eta-1)] \tag{6}$$

where the pressure jump factor y is given by

$$y = P_b/P_a \tag{7}$$

Equations (4) and (6) are known as the mechanical shock conditions since they are independent of the fluid through which the shock is propagating. Other useful relations obtained directly from these conditions are

$$V_a^2 = (P_o/\rho_a)(y-1) \ [\eta/(\eta-1)] \tag{8a}$$

$$(V_a-V_b)^2 = (P_a/\rho_a) \ (y-1) \ [(\eta-1)/\eta] \tag{8b}$$

To proceed further, the thermodynamic properties of the medium through which the shock is moving must be considered.

Solution For Air

Equation (1c), which expresses conservation of energy across the shock, becomes, on utilizing the thermodynamic constraint defined by equation (2),

$$[\gamma/(\gamma-1)](P_a/\rho_a) + V_a^2/2 = [\gamma/(\gamma-1)](P_b/\rho_b) + V_b^2/2 \tag{9}$$

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On substituting equations (6) and (8a) and dividing through by P_a/ρ_a equation (9) yields

$$\frac{\gamma}{\gamma-1} + \frac{\eta(y-1)}{2(\eta-1)} = \left(\frac{\gamma}{\gamma-1}\right) \frac{y}{\eta} + \frac{y-1}{2\eta(\eta-1)} \qquad (10)$$

Some simple algebraic manipulation of this equation gives the relation between the compressibility factor η and the pressure jump factor y for an air shock as

$$\eta = (M-y)/(1 + My) \qquad (11a)$$

where

$$M = (\gamma-1)/(\gamma + 1) \qquad (11b)$$

Solution for Water

For a water shock, the relation between η and y is obtained directly from the Tait equation of state, equation (3a), as

$$\eta = [(y + \beta)/(1 + \beta)]^{1/n} \qquad (12a)$$

where

$$\beta = (B - P_o) / P_a \qquad (12b)$$

Shock Refraction at an Air-Water Interface

The refraction at an air-water interface of a plane shock wave incident through the air is schematically illustrated in Figure 1.

From Figure 1, the relative velocities of the fluids as seen from the shocks can readily be ascertained. For the refracted water shock, these relative velocities are:

$$V_a = -U'' \qquad (13a)$$

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$$v_b = u_2 - U'' \qquad (13b)$$

Hence,

$$U'' = -v_a \qquad (14a)$$

$$u_2 = v_a - v_b \qquad (14b)$$

On substituting from equations (8) and noting that $\rho_a = \rho_{wo}$ and $P_a = P_o$, the water shock and particle velocities are obtained as

$$U'' = \left\{ (P_o/\rho_{wo}) \ (y - 1) \ [\eta_w/(\eta_w - 1)] \right\}^{1/2} \qquad (15a)$$

$$u_2 = \left\{ (P_o/\rho_{wo}) \ (y-1) \ [(\eta_w - 1)/\eta_w] \right\}^{1/2} \qquad (15b)$$

where

$$y = P_2/P_o \qquad (16)$$

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The water shock compressibility factor η_w is found from equation (12a) as

$$\eta_w = \rho_{w2}/\rho_{wo} = \left[1 + n\,\alpha(y-1) \right]^{1/n} \tag{17}$$

where

$$\alpha = P_o/nB = P_o/\rho_{wo}C_{wo}^2 \tag{18}$$

and where equation (12b) has been used to obtain

$$\beta = B/P_o - 1 = 1/n\,\alpha - 1 \tag{19}$$

Similarly, for the incoming air shock, the shock front and particle velocities are calculated from equations (8) and (11a) as

$$U = \left\{ (P_o/\rho_{ao}) \left[(x+M)/(1-M) \right] \right\}^{\frac{1}{2}} \tag{20a}$$

$$u_1 = (x-1) \left\{ (P_o/\rho_{ao}) \left[(1-M)/(x+M) \right] \right\}^{\frac{1}{2}} \tag{20b}$$

where

$$x = P_1/P_o \tag{21}$$

For the refracted air shock, the relative velocities with respect to the shock are seen, from Figure 1, to be

$$v_a = u_1 - U' \tag{22a}$$

$$v_b = u_2 - U' \tag{22b}$$

Hence,

$$v_a - v_b = u_1 - u_2 \tag{23}$$

where u_1 and u_2 are given by equations (20b) and (15b), respectively. However, from equations (8b) and (11a), $v_a - v_b$ can also be written as

$$v_a - v_b = (z-1) \left\{ (P_1/\rho_{a1}) \left[(1-M)/(z+M) \right] \right\}^{\frac{1}{2}} \tag{24a}$$

where

$$z = P_2/P_1 = (P_2/P_o)/(P_1/P_o) = y/x \tag{24b}$$

Equation (23) then becomes, on substituting the appropriate expressions,

$$(z-1)\left(\frac{P_1}{\rho_{a1}}\right)^{\frac{1}{2}} \left(\frac{1-M}{z+M}\right)^{\frac{1}{2}} = (x-1)\left(\frac{P_o}{\rho_{ao}}\right)^{\frac{1}{2}} \left(\frac{1-M}{x+M}\right)^{\frac{1}{2}} - \left(\frac{P_o}{\rho_{wo}}\right)^{\frac{1}{2}} (y-1)^{\frac{1}{2}} \left(\frac{\eta_w-1}{\eta_w}\right)^{\frac{1}{2}} \quad (25)$$

where η_w is given by equation (17). Upon multiplying through by $(\rho_{ao}/P_o)^{\frac{1}{2}}$ substituting $z=y/x$, and noting from equation (11a) that

$$\rho_{a1}/\rho_{ao} = (M + x) / (1 + Mx) \quad (26)$$

equation (25) yields

$$(y-x)\left[\frac{(1-M)(1+Mx)}{(y+Mx)(M+x)}\right]^{\frac{1}{2}} = (x-1)\left(\frac{(1-M)}{M+x}\right)^{\frac{1}{2}} - \left(\frac{\rho_{ao}}{\rho_{wo}}\right)^{\frac{1}{2}} (y-1)^{\frac{1}{2}} \left(\frac{\eta_w-1}{\eta_w}\right)^{\frac{1}{2}} \quad (27)$$

The solution of equation (27) gives the refracted pressure ratio y ($=P_2/P_o$) in terms of the incoming pressure ratio $x(=P_1/P_o)$.

Comparison Between The In-Water Shock Waves Obtained From Exact Shock And Acoustic Theories

The process of shock refraction produces at the water surface a step change in pressure with a magnitude given by $P_2 - P_o = P_o (y-1)$. The response of the water, treated as a linear acoustic medium, to this step change in surface pressure is well known [3]. Retaining the previous notation and denoting values predicted by acoustic theory with a superscript "*", the propagation velocity U''^* of the step (shock front) into the water is given by

$$U''^* = C_{wo} \quad (28)$$

Behind this front, the water particle velocity u_2^* and the water density ρ_{w2}^* are found, respectively, as

$$u_2^* = (P_o/\rho_{wo} \, C_{wo}) (y-1) \quad (29)$$

and

$$\rho_{w2}^*/ \rho_{wo} = 1 + \alpha(y-1) \quad (30)$$

with α defined by equation (18).

The exact shock and linear acoustic predictions of the water response to the refracted pressure wave P_2 can be compared by forming the ratios U''/U''^*, U_2/U_2^*, and ρ_{w2}/ρ_{w2}^*. From equations (15), (17), (28), (29), and (30), these ratios are determined as

$$U''/U''^* = \left\{ \frac{Q (1 + nQ)^{1/n}}{(1 + nQ)^{1/n} - 1} \right\}^{\frac{1}{2}} \quad (31a)$$

$$U_2/U_2^* = \left\{ \frac{(1 + nQ)^{1/n} - 1}{Q (1 + nQ)^{1/n}} \right\}^{\frac{1}{2}} \quad (31b)$$

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$$\rho_{w2}/\rho_{w2}^{*} \quad = \quad \frac{(1 + nQ)^{1/n}}{1 + Q} \tag{31c}$$

where

$$Q = \alpha \ (y-1) \tag{31d}$$

The above ratios are plotted in Figure 2 as a function of the refracted pressure ratio $y = P_2/P_o$. In obtaining these curves, values of $n=7.15$ (ref. [21]) and $\alpha = 4.5 \times 10^{-5}$ ($P_o = 1.013 \times 10^{6}$ dynes/cm^2, $\rho_{wo} = 1.0$ gm/cm^3, $C_{wo} = 1.5 \times 10^{5}$ cm/sec) were used. For refracted pressures up to 1200 atmospheres, Figure 2 shows that the in-water shock front and particle velocities calculated by exact shock theory differ by less than 10% from the in-water shock front and particle velocities predicted by acoustic theory. Figure 2 also shows that the changes in density predicted by exact shock and acoustic theories differ by less than 1% for refracted pressures up to 1200 atmospheres.

Comparison Between the Refracted Pressure and the Pressure Reflected From a Rigid Wall

For an incoming shock of strength $x=P_1/P_o$ (see Figure 1), the pressure P_{2R} reflected from a rigid wall is given by [1].

$$y_R = P_{2R}/P_o = x \ [(1+2M) \ x-M]/ \ (1+Mx) \tag{32}$$

For the same incoming shock strength, the pressure P_2 refracted at the air-water interface is found as the solution of equation (27).

To compare the refracted pressure to the pressure reflected at a rigid wall, the ratio P_2/P_{2R} is formed. This ratio is plotted in Figure 3 as a function of the reflected shock strength P_{2R}/P_o. (The numbers in parenthese below the abscissa correspond to the incoming shock strength P_1/P_o required to yield the reflected shock strength P_{2R}/Po.) In obtaining this curve, values of $M=1/6$ ($\gamma =1.4$) and $\rho_{ao} = 1.2 \times 10^{-3}$ gm/cm^3 were used. Figure 3 clearly shows that the ratio of the pressure refracted at an air-water interface to the pressure reflected from a rigid wall remains above 96% for reflected pressure up to 1200 atmospheres (incoming pressure up to 156 atmospheres).

CONCLUSIONS

In-water shock front velocity U, particle velocity u and density ρ can be predicted by acoustic theory up to 1200 atm with a maximum error of 10% at the high pressures.

The refracted pressure can be calculated by treating the surface as a rigid wall up to 1200 atm with maximum error of 4% at 1200 atm.

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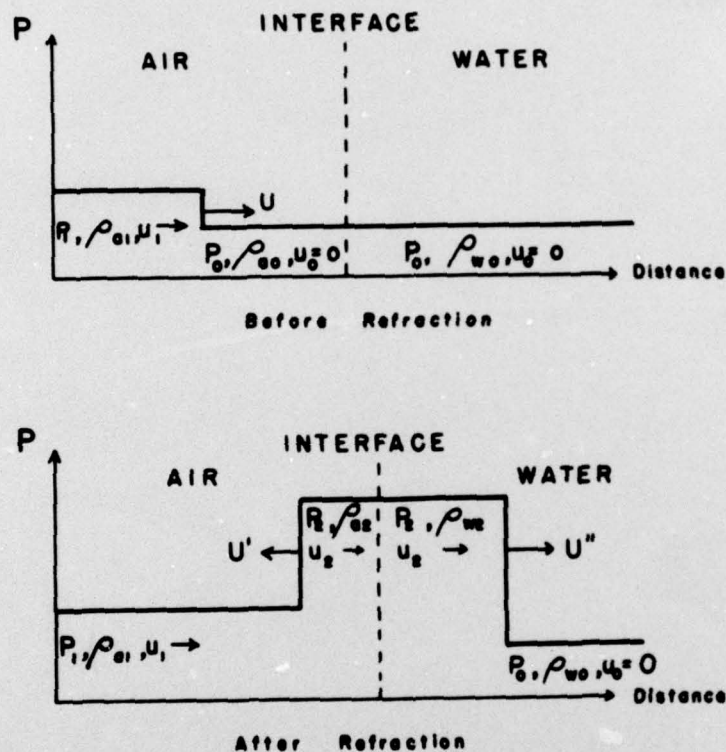
P | **INTERFACE**

AIR | WATER

$P_1, \rho_{a1}, u_1 \rightarrow$ $\overset{\longrightarrow}{U}$

$P_0, \rho_{a0}, u_0 = 0$ $P_0, \rho_{w0}, u_0 = 0$ Distance

Before Refraction

P | **INTERFACE**

AIR | WATER

$U' \leftarrow$ $\begin{array}{cc} P_2, \rho_{a2} & P_2, \rho_{w2} \\ u_2 \rightarrow & u_2 \rightarrow \end{array}$ $\rightarrow U''$

$P_1, \rho_{a1}, u_1 \rightarrow$

$P_0, \rho_{w0}, u_0 = 0$ Distance

After Refraction

Fig. 1 — Schematic illustration of the refraction of a plane shock at an air-water interface. The symbols U and u denote, respectively, the shock front and particle velocities with respect to a fixed frame.
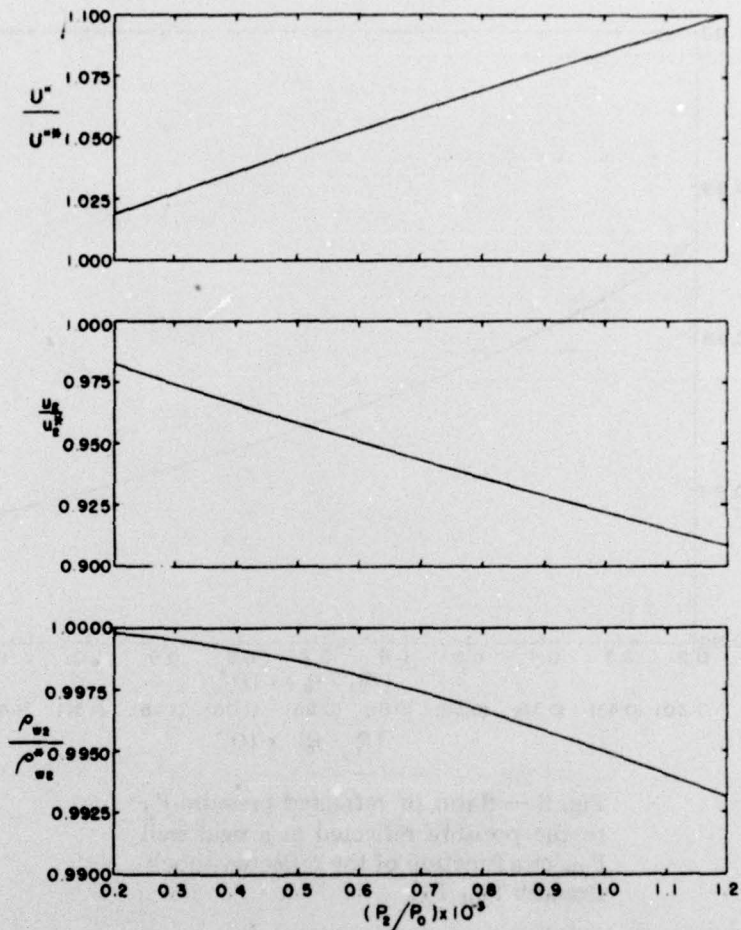
Fig. 2 — Comparison of exact shock (this paper) to acoustic theory for water shockwave velocity U'', particle velocity u_2 and water density ρ_{w2}. The * denotes acoustic values. Comparative values are plotted as a function of the refracted pressure ration P_2/P_o.
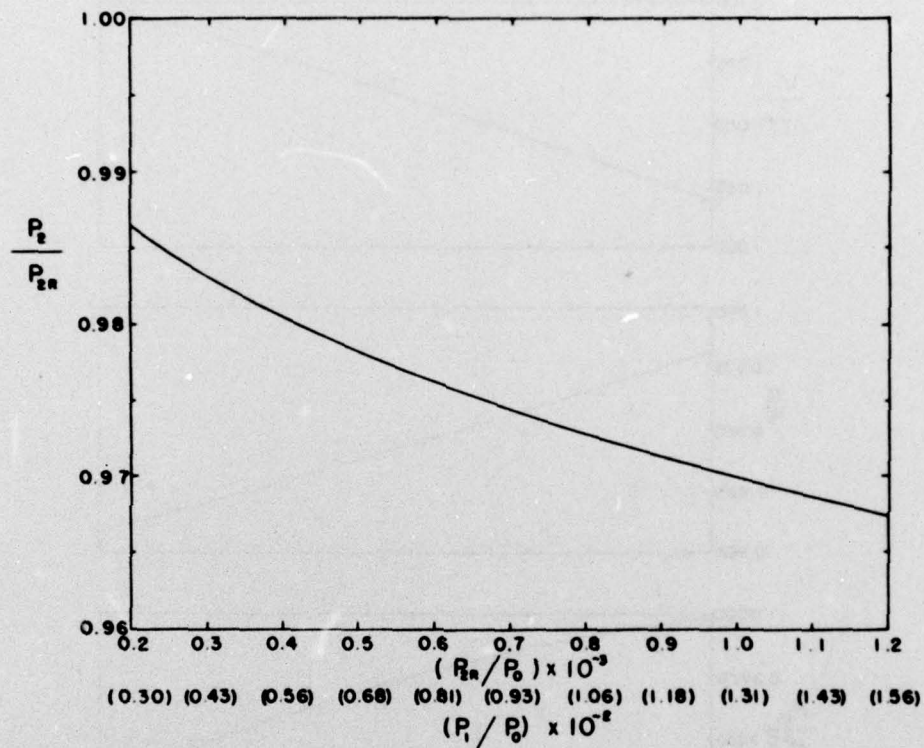
Fig. 3 — Ratio of refracted pressure P_2
to the pressure reflected at a rigid wall
P_{2R} as a function of the reflected shock
strength P_{2R}/P_o.